An improved elasto-visco-plastic constitutive model for snow

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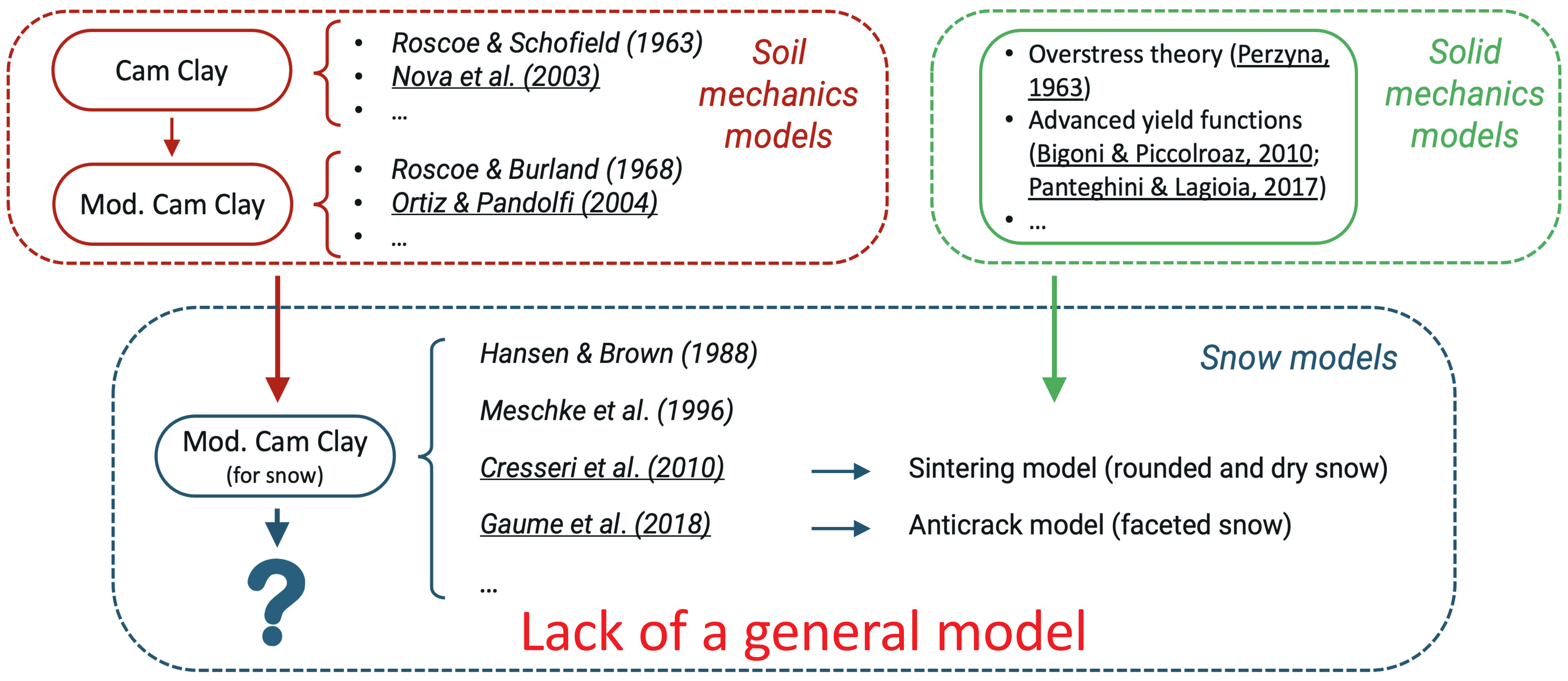
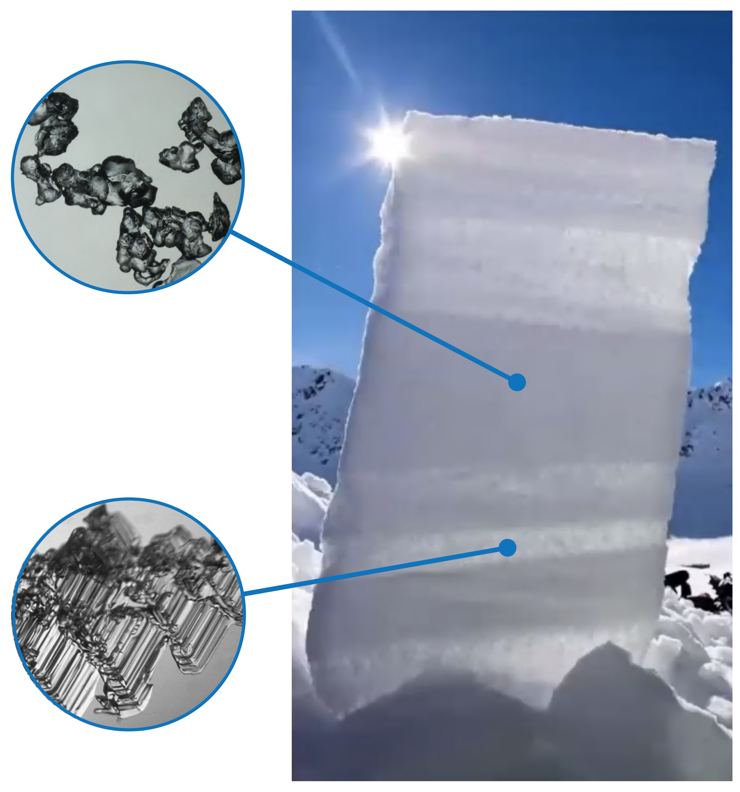
1. Introduction

Snow is a granular, 3-phase, and high-porosity material whose macroscopic behaviour is strongly influenced by **microstructural processes** (e.g., sintering, metamorphisms and shape of grains).

Traditionally, available snow models derive from models originally conceived for soils.

For continuum mechanics purposes the Modified Cam-Clay (MCC) model is the most used.



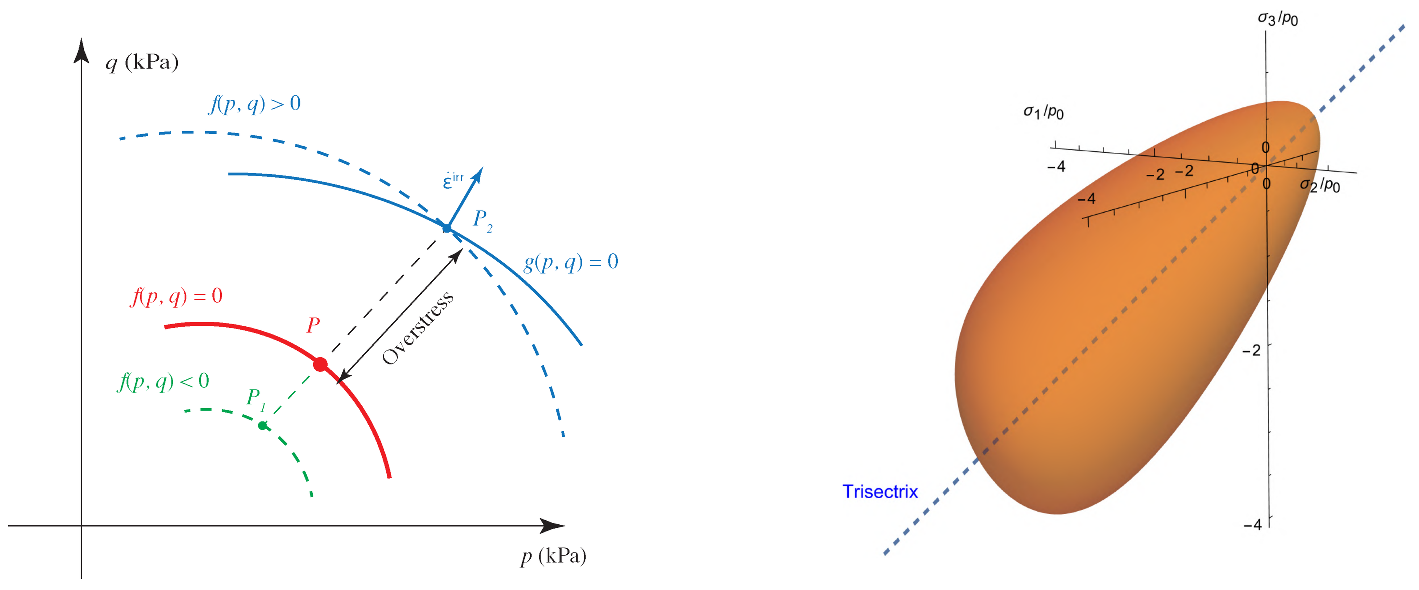


2. The improved model

Initial hypotheses: small strains, continuity, homogeneity, isotropy and isothermal conditions during the test.

The improved model is based on **three key points**:

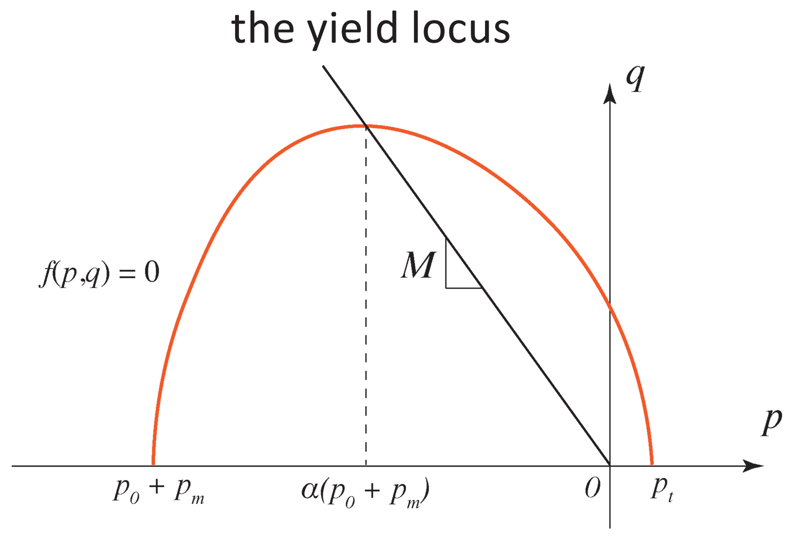
1. the general framework of the elasto-visco-plastic model by **Cresseri et al. (2010);**
2. the **overstress theory of Perzyna (1963)**, accounting for irrecoverable strains even inside the elastic region;
3. a new formulation for an **asymmetrical yield function.**



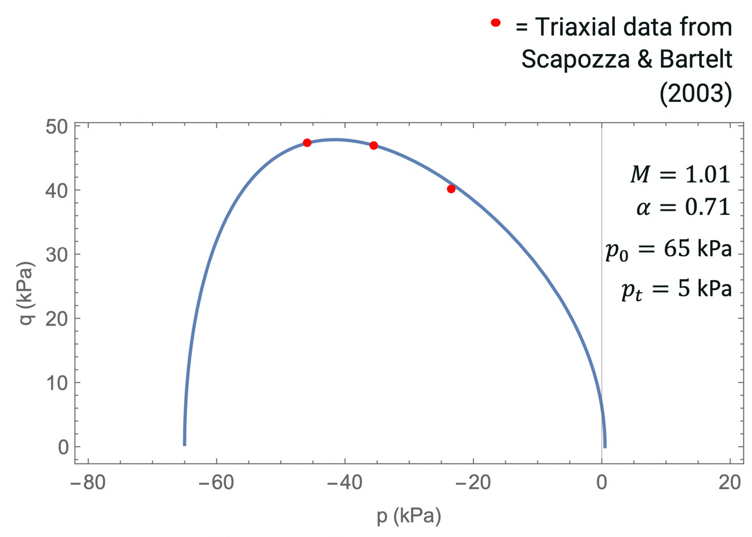
The new yield function is introduced to account for an asymmetrical, always smooth and simply convex yield locus:

$$f(p,q)=\frac{1}{p_{atm}^2}\left\{q^2-4\alpha^2M^2(p_0+p_m)^3\frac{(\alpha-1)(p-p_t)(p+p_0+p_m)[p_t+\alpha(p_0+p_m)]}{\{-p(p_0+p_m-p_t)+2p(p_0+p_m)\alpha+(p_0+p_m)[-(\alpha-2)p_t+(p_0+p_m)\alpha]\}^2}\right\}$$

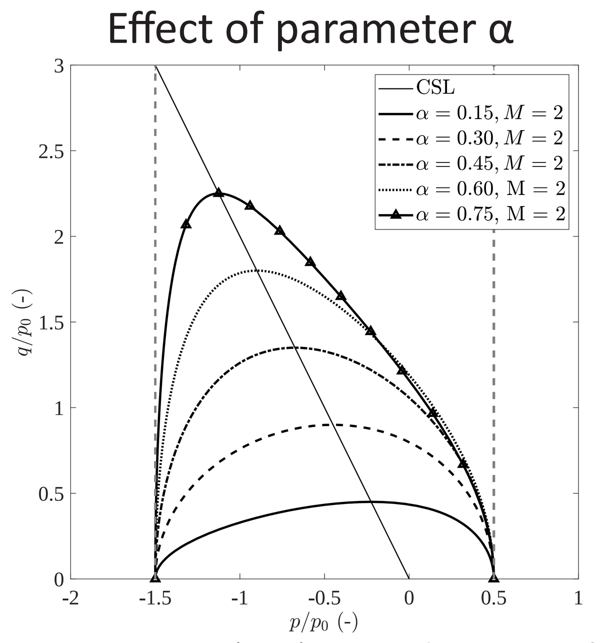
Sketch of the meridian section of the yield locus



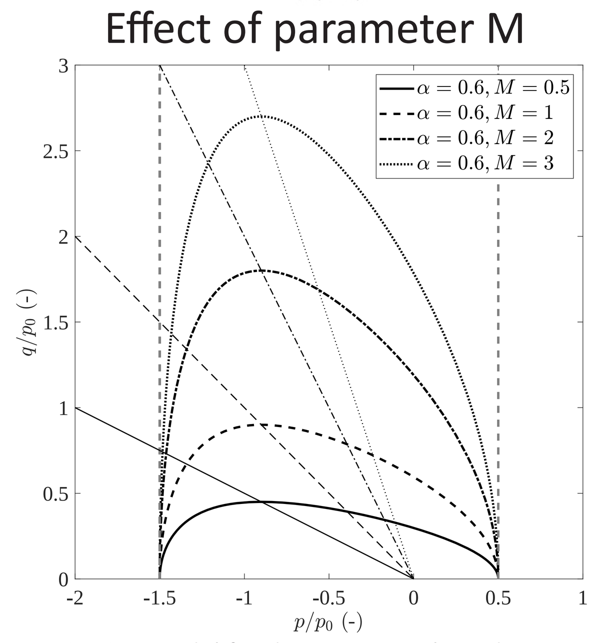
Fit with experimental data



Effect of parameter α



Effect of parameter M



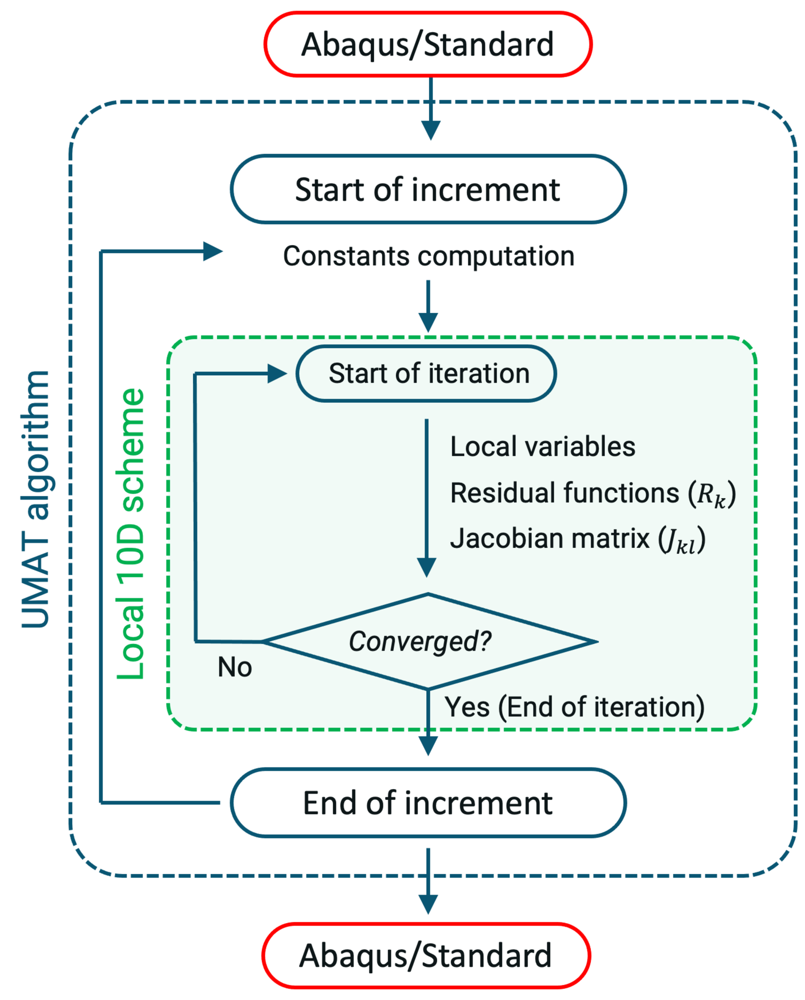
Cresseri S, Genna F, Jommi C (2010) Numerical integration of an elastic–viscoplastic constitutive model for dry metamorphosed snow. Int J Num Anal Met Geomech 34(12), 1271-1296

Panteghini A & Lagioia R (2018) An extended modified Cam-Clay yield surface for arbitrary meridional and deviatoric shapes retaining full convexity and double homothety. Géotechnique 68(7), 590-601

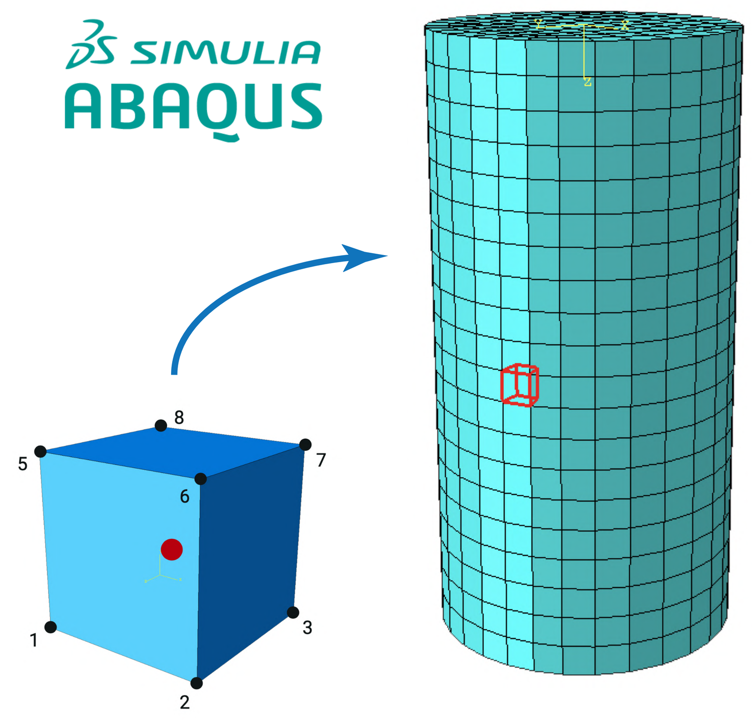
3. Numerical implementation

The model is time-integrated with a **fully implicit backward Euler method**.

The local problem (solving the system of 10 nonlinear differential constitutive equations) is solved with a Powel hybrid method.



SIMULIA ABAQUS

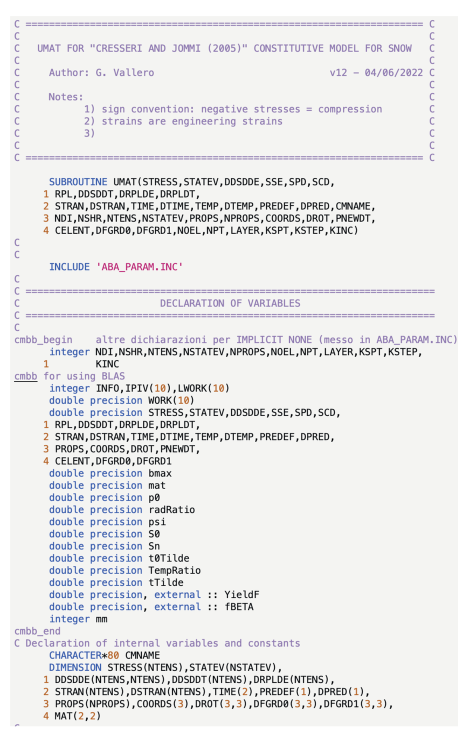


The model has been implemented into the **UMAT format** (Fortran 77) for the Abaqus/Standard Finite Element Code.

First results were obtained with elliptical yield surface and the free code IncrementalDriver.

Local system of 10 differential constitutive equations to be solved for all time increments at any integration point:

$$\begin{cases} R_{1to6}=\frac{1}{Z_1}\left(\Delta\boldsymbol{\sigma}-\boldsymbol{D}^e(\boldsymbol{\sigma}_{n+1})\Delta\boldsymbol{\epsilon}+\boldsymbol{D}^e(\boldsymbol{\sigma}_{n+1})\,\beta_{n+1}\left.\frac{\partial g}{\partial\boldsymbol{\sigma}}\frac{1}{|\nabla g|}\right|_{n+1}\Delta t\right)=0\\ R_7=\frac{1}{Z_2}\left(g(\boldsymbol{\sigma}_{n+1},p_{g0})\right)=0\\ R_8=\frac{1}{Z_1}\left(\Delta p_0+\frac{v_{n+1}}{\lambda-k}(p_{0n}+\Delta p_0)\Delta\epsilon_v^{irr}\right)=0\\ \mathrm{R}_9=\Delta p_m-\pi_m b_{max}\Delta S=0\\ R_{10}=S_n+\Delta S\\ \quad-\tilde{S}_0(t_s,r,T)\left\{1-\tanh\left[C\left(\sum_{i=0}^{n-1}\left(\int_{t_i}^{t_i+\Delta t_i}\sqrt{\left(\frac{\epsilon_{v_i}^{irr}}{\Delta t_i}\right)^2+\left(\frac{\epsilon_{dev_i}^{irr}}{\Delta t_i}\right)^2}\,dt\right)+\int_{t_n}^{t_{n+1}}\sqrt{\left(\frac{\epsilon_v^{irr}}{\Delta t}\right)^2+\left(\frac{\epsilon_{dev}^{irr}}{\Delta t}\right)^2}\,dt\right)\right]\right\}=0 \end{cases}$$



Some results obtained on the basis of available literature data:

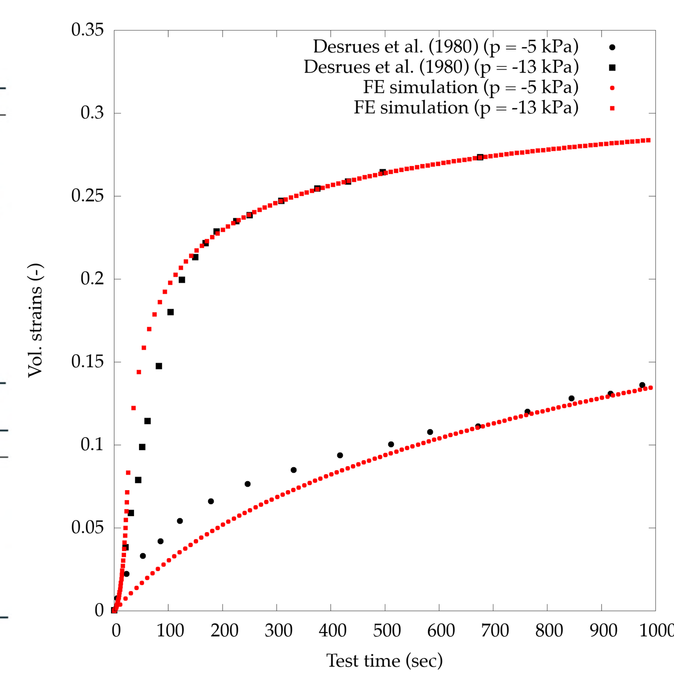
Volumetric creep tests

Constitutive parameters

Parameter	Value
λ	0.35
k	0.02
G	4000 kPa
γ	0.0000042
A	16
π_m	40
$\chi = \chi_g$	0.1
C	0.01
ω	0.05
$M_{ch} = M_{eh}$	0.88

Initial conditions

Parameter	Value
v^0	2.88
r^0	0.2 mm
p_0	77 kPa
p^0	-66 kPa
T^0	-5 °C
t_s^0	0 min



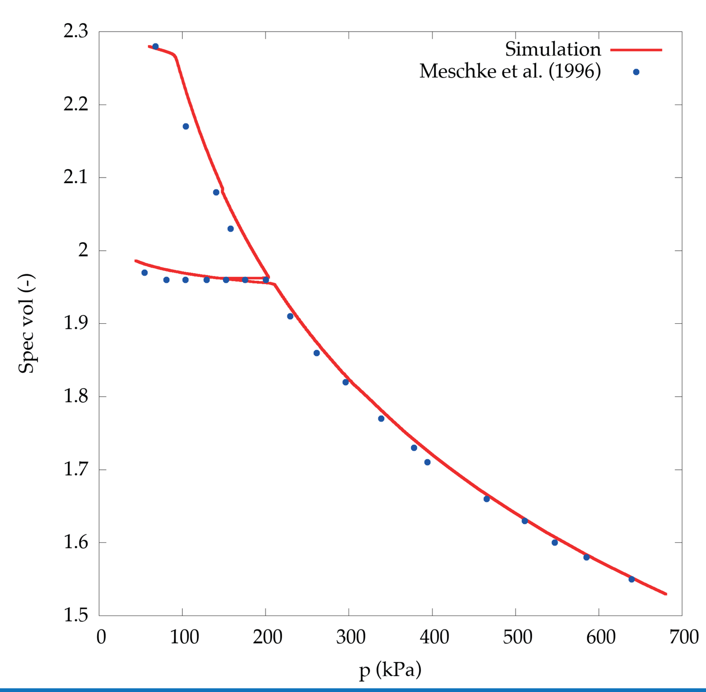
Volumetric compression test

Constitutive parameters

Parameter	Value
λ	0.35
k	0.02
G	2114 kPa
γ	0.00001
A	16
π_m	40
$\chi = \chi_g$	0.1
C	0.01
ω	0.05
$M_{ch} = M_{eh}$	0.88

Initial conditions

Parameter	Value
v^0	4.58
r^0	0.2 mm
p_0	0.1 kPa
p^0	-0.01 kPa
T^0	-5 °C
t_s^0	0 min



Oral presentations / Publications (last year)

- Vallero G. et al. An improved asymmetrical yield surface for the elasto-visco-plastic modelling of snow (in preparation)
- Vallero G. et al. (2022) Experimental study of the shear strength of a snow-mortar interface. Cold Reg Sci Technol 193
- Vallero G. et al. (2022) Some Computational Issues in the Elasto-Plastic Modelling of Snow. Proc. of COMPLAS 2021
- Vallero G. et al. (2022) Advances in Constitutive Modelling of Snow for Finite Element Analysis. AIMETA 2022, Palermo (Italy), 4-8 September 2022
- Vallero G. et al. (2022) New perspectives for the elasto-plastic modelling of snow. WCCM-APCOM 2022, online, 31 July - 5 August 2022

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